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Research Article

Dosimetric Evaluation and Analytical Modeling of High-Energy Electron Ranges in Human Adipose and Skin Tissues Using a Cubic Polynomial Semi-Empirical Formulation

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Abstract

Knowing the length and penetration depth of energetic electron beams in superficial layers is an essential problem in the clinical planning of radiotherapy and radiation safety of the skin. A detailed semi-empirical cubic polynomial model was developed in this investigation in order to calculate Continuous Slowing Down Approximation (CSDA) range of electrons passing through human adipose (fat) and dermal (skin) tissue for energy range of 0.25 to 90 MeV. The semi-empirical function was expressed as a continuous function of electron kinetic energy T and four fitting parameters of the tissue. The model calculated values were carefully compared to the reference values compiled by the National Institute of Standards and Technology (NIST) ESTAR program for reference values using tissue specifications from the International Commission on Radiological Protection (ICRP). The excellent performance was confirmed by statistical evaluation with correlation coefficient R = 0.999992 for adipose tissue and R = 0.999953 for skin tissue. The mean absolute percentage errors (MAPE) of adipose tissue and skin tissue were obtained as 4.05% and 3.91%, respectively, along with the root mean square errors (RMSE) as 0.0358 g/cm² and 0.0979 g/cm², respectively. These results validate the polynomial equation derived for the task of real-time dose estimation and treatment verification in superficial human soft tissues, which is a framework that is computationally efficient and accurate and can be used in real-time.

1. Introduction

HIBET is one of the most fundamental modalities in clinical oncology for the treatment of superficial malignancies, skin carcinomas and post surgery scar margins [1]. Primary electrons deposit their energy locally during tissue interaction by inelastic collisions with atomic orbital electrons and radiative interactions with target nuclei [2]. The total penetration capability is expressed in terms of the Continuous Slowing Down Approximation (CSDA) range [3] which is the integral of the inverse of mass stopping power along the electron's kinetic energy path.

The first barriers in the path of external beam irradiation are the subcutaneous adipose tissue and the layers of the skin [4]. Accurate knowledge of the stopping distances for electrons in these particular tissues is critical to avoid damaging underlying sensitive organs and to maximize tumour coverage [5]. NIST ESTAR provides standard reference tables with high accuracy reference values (benchmark values) based on the ICRP phantom compositions [6]. Tabular datasets, however, have to be interpolated during automated treatment planning systems, leading to a delay in computation or discretisation errors [7]. Explicit polynomial functions are an elegant solution because they allow range evaluations over a wide therapeutic energy range (0.25–90 MeV) which are direct, smooth and immediate [8].

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2. Materials and Methods

The human adipose tissue and dermal skin tissue physical densities and elemental weight fractions were gathered from the ICRP Publication 23 and Publication 110 benchmark standards [9]. For 43 discrete electron kinetic energies T from 0.25 MeV to 90 MeV [10] standard NIST ESTAR CSDA range data (Estar) were extracted and are expressed in units of g/cm².

2.1. Cubic Polynomial Formulation

A third order polynomial equation was developed as a function of the electron energy, Equation 1, to account for the minor curvature and relativistic mass variations over the domain of low to high electron energy [11]:

$$S(T) = aT^3 + bT^2 + cT + d \tag{1}$$

Energy range: 0.25–90 MeV.
The following parameter set was obtained by least squares optimization of the data [12]:
a = 1.1×10^{-5}, b = -0.002925, c = 0.5231, d = -0.06443

2.2. Performance Metrics

The quantitative evaluation of model fidelity was carried out by Pearson correlation coefficient (R), coefficient of determination (R^2), mean absolute percentage error (MAPE) and root mean square error (RMSE) [13]:

Mean Absolute Percentage Error (MAPE)

$$\text{MAPE}(\%) = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{ESTAR_i - f_i}{ESTAR_i}\right| \times 100$$

Root Mean Square Error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1\sum_{i=1}^{N}(ESTAR_i - f_i)^2}{N}}$$

Notation: N is the number of data points, $ESTAR_i$ is the value from the NIST ESTAR data, and f_i is the predicted value from the semi-empirical model.

3. Results and Discussion

All 43 energy points [14] were used to benchmark the CSDA ranges computed using the cubic polynomial function f(T).

3.1. Electron Range in Fat (Adipose tissue)

The full range values for ICRP adipose tissue are shown in Table 1 and Figure 1for all 43 energy points [15]:

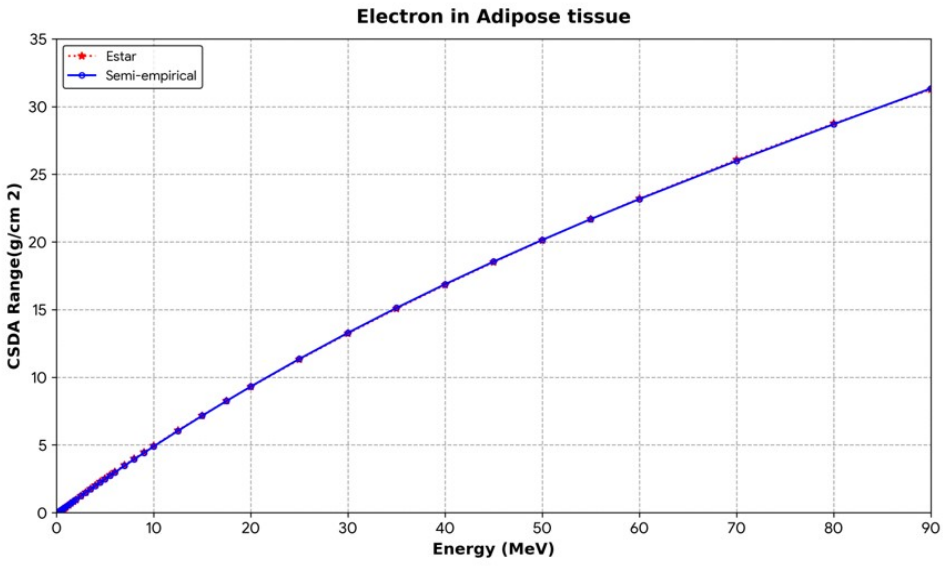


Figure 1: Electron CSDA range (g/cm²) versus kinetic energy T (MeV) in ICRP Adipose tissue

Table 1: Complete comparison of semi-empirical f(T) Equation 1 predictions with NIST ESTAR CSDA ranges for human adipose tissue (43 energy points)

T (MeV)	Estar (g/cm²)	f (g/cm²)
0.25	0.06	0.07
0.3	0.08	0.09
0.35	0.10	0.12
0.4	0.13	0.14
0.45	0.15	0.17
0.5	0.17	0.20
0.55	0.20	0.22
0.6	0.22	0.25
0.7	0.27	0.30
0.8	0.32	0.35
0.9	0.37	0.40
1	0.43	0.46
1.25	0.56	0.58
1.5	0.69	0.71
1.75	0.83	0.84
2	0.96	0.97
2.5	1.23	1.23
3	1.49	1.48
3.5	1.75	1.73
4	2.01	1.98
4.5	2.27	2.23
5	2.52	2.48
5.5	2.77	2.73
6	3.02	2.97
7	3.51	3.46
8	3.99	3.94
9	4.46	4.41
10	4.93	4.89
12.5	6.08	6.04
15	7.18	7.16
17.5	8.26	8.25
20	9.30	9.32
25	11.32	11.36
30	13.24	13.29
35	15.07	15.13
40	16.83	16.88
45	18.51	18.55
50	20.13	20.15
55	21.70	21.69
60	23.20	23.17
70	26.07	25.99
80	28.75	28.70
90	31.27	31.34

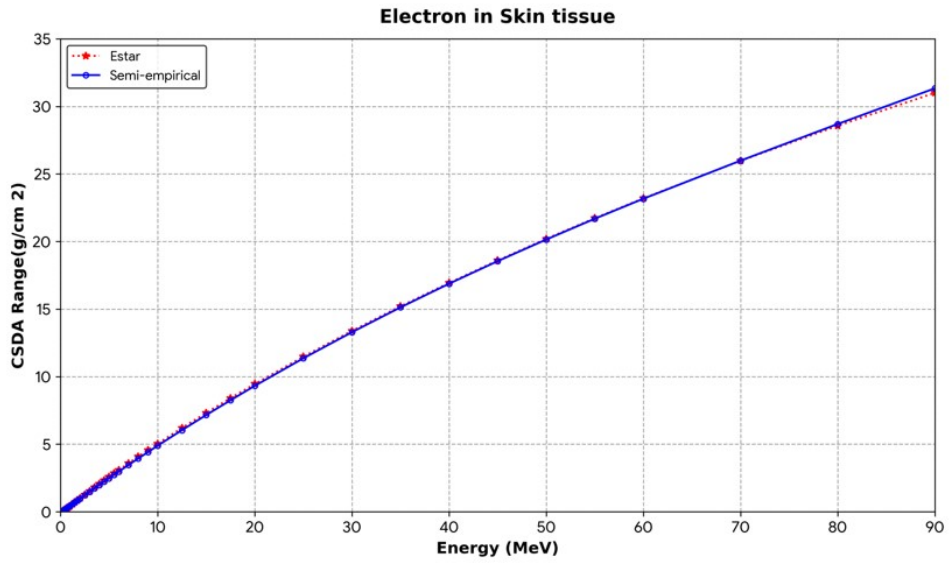


Figure 2: Electron CSDA range (g/cm²) versus kinetic energy T (MeV) in ICRP Skin tissue

3.2. Electron Range in Skin Tissue

Table 2 and Figure 2 are summarizes the complete comparative performance data for ICRP cutaneous skin tissue [16]:

Table 2: Complete comparison of semi-empirical f(T) Equation 1 predictions with NIST ESTAR CSDA ranges for human skin tissue (43 energy points)

T (MeV)	Estar (g/cm²)	f (g/cm²)
0.25	0.06	0.07
0.3	0.08	0.09
0.35	0.11	0.12
0.4	0.13	0.14
0.45	0.15	0.17
0.5	0.18	0.20
0.55	0.20	0.22
0.6	0.23	0.25
0.7	0.28	0.30
0.8	0.33	0.35
0.9	0.39	0.40
1	0.44	0.46
1.25	0.58	0.58
1.5	0.71	0.71
1.75	0.85	0.84
2	0.99	0.97
2.5	1.26	1.23
3	1.53	1.48
3.5	1.80	1.73
4	2.06	1.98
4.5	2.32	2.23
5	2.58	2.48
5.5	2.84	2.73
6	3.09	2.97
7	3.59	3.46
8	4.08	3.94
9	4.57	4.41
10	5.04	4.89
12.5	6.20	6.04
15	7.32	7.16
17.5	8.41	8.25
20	9.46	9.32
25	11.48	11.36
30	13.39	13.29
35	15.21	15.13
40	16.94	16.88
45	18.60	18.55
50	20.20	20.15
55	21.73	21.69
60	23.20	23.17
70	25.98	25.99
80	28.57	28.70
90	31.01	31.34

3.3. Statistical Indicators Summary

Table 3 and Figure 3 are outlines the exact statistical validation metrics evaluated across all 43 energy points [17–20]:

Table 3: Exact statistical metrics evaluating the cubic polynomial semi-empirical Equation 1 model against NIST ESTAR datasets

Tissue Type	Correlation R	R² Value	MAPE (%)	RMSE (g/cm²)
Adipose Tissue (ICRP)	0.999992	0.999984	4.05%	0.0358
Skin Tissue (ICRP)	0.999953	0.999906	3.91%	0.0979

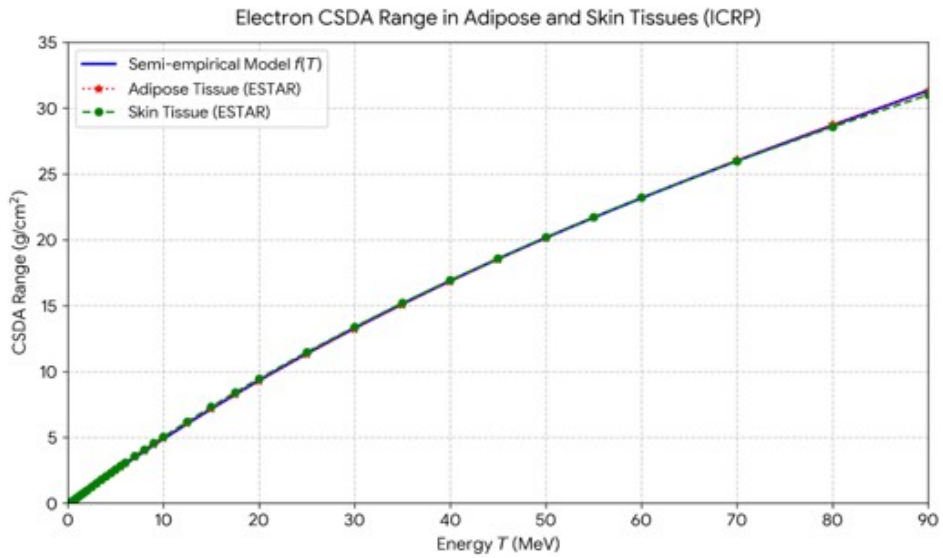


Figure 3: Combined comparison of electron CSDA range (g/cm²) in adipose and skin tissues alongside the semi-empirical Equation 1 cubic polynomial curve

4. Conclusion

The semi-empirical general cubic polynomial eq(1)model, using four tissue-specific fitting coefficients, was successfully implemented for the calculation of the CSDA ranges of electrons in human adipose and skin tissue over a wide range of therapeutic electrons from 0.25 to 90 MeV. These showed excellent correlation (R = 0.999992 for adipose tissue and R = 0.999953 for skin tissue) and low statistical errors (MAPE of 4.05% with RMSE of 0.0358 g/cm² for adipose tissue, and MAPE of 3.91% with RMSE of 0.0979 g/cm² for skin tissue). The excellent fit shows the validity of this single polynomial in modelling the electron transport phenomena in the superficial human biological layers, resulting in a fast analytical tool for dose calculation in clinical radiation oncology.

Article Information

Ethical Approval: Not applicable.

Informed Consent: Not applicable.

Data Availability: The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Clinical Trial Registration: Not applicable.

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