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Research Article

Formulation and Validation of a Generalized Semi-Empirical Equation for Calculating Electron CSDA Range in Human Tissues (Brain and Eye Lens) for Radiation Therapy Dosimetry

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Abstract

The accurate determination of the range of high-energy electrons in human biological tissues is of great significance in radiation therapy planning and clinical radiation protection. In this work, we developed a single semi-empirical formula for the Continuous Slowing Down Approximation (CSDA) range of electrons in human tissues with special attention to brain and eye lens tissues and used the International Commission on Radiological Protection (ICRP) reference standards for the energy range 100-550 MeV. The semi-empirical non-linear power-law expression $f(T) = (a \times T^b) + c$ is parameterized using constants a = 34.12, b = 0.2348 and c = -67.5. The semi-empirically predicted ranges (f) were systematically compared to the standard reference values (SRV) obtained from the National Institute of Standards and Technology (NIST) ESTAR database (Estar). An exceptional correlation coefficient of R = 0.99998 (R^2 = 0.99996) was found for both brain and eye lens tissues in the statistical analysis. The mean absolute percentage error (MAPE) was very low at 0.177% for the eye lens and 1.652% for brain tissue, whereas, the root mean square error (RMSE) was very low at 0.108 g/cm^2 for eye lens and 1.034 g/cm^2 for brain tissue. Importantly, the results demonstrate that this single semi-empirical formulation is applicable and can be used in radiotherapy to accurately predict the range of electrons in all soft tissues in the human body, offering an efficient, accurate and real-time analytical tool to replace the time-consuming Monte Carlo simulation.

1. Introduction

Relativistic electrons undergo both ionization and excitation processes, and they generate radiation energy through the radiative bremsstrahlung process, when they interact with the atomic electrons and atomic nuclei in high energy radiotherapy. The Continuous Slowing Down Approximation (CSDA) range is the average length of electron's path from its original kinetic energy to its final rest [1].

During cranial radiotherapy, it is important to have an accurate estimation of electron ranges in anatomical structures that have been specified by ICRP phantom properties, especially for the brain and the eye lens [2]. The eye lens is highly sensitive to radiation-induced cataractogenesis [3] and brain tissue contains important functional regions that need a careful dosimetric assessment [4]. While NIST ESTAR tables are the international gold standard [5], explicit semi-empirical mathematical relationships can be used to quickly and directly calculate electron ranges across continuous energy intervals (100-550 MeV) without the need to use multi-dimensional lookup tables [6]. Additionally,

because of the slight differences in the effective atomic numbers of the various soft human organs, a semi-empirical general equation, eq(1), can be used to model the electron transport in all human soft tissues during radiation treatment [7].

2. Materials and Methods

Human brain and eye lens tissues elemental compositions and mass densities were set according to the recommendations of ICRP Publication 23 and 110 [8]. The reference values for kinetic energy T in the range of 100 to 550 MeV of the NIST ESTAR CSDA [9] were used.

2.1. Derived Semi-Empirical Formulation

An explicit power-law offset equation (eq(1); [10]) was derived to model the non-linear relationship between kinetic energy T (MeV) (100-550MeV) and CSDA range f (g/cm^2).

$$f(T) = (a \times T^b) + c \quad 100 \leq T \leq 550MeV \tag{1}$$

The optimum parameters for human tissue calculation were set by non-linear regression analysis [11]:

a = 34.12, b = 0.2348, c = -67.5

2.2. Statistical Evaluation Metrics

The agreement between the semi-empirical eq(1) model and the benchmark values from NIST ESTAR was quantified using the correlation coefficient (R), coefficient of determination (R^2), mean absolute percentage error (MAPE) and root mean square error (RMSE) [12]:

Mean Absolute Percentage Error (MAPE)

$$\text{MAPE}(\%) = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\text{ESTAR}_i - f_i}{\text{ESTAR}_i}\right| \times 100$$

Root Mean Square Error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1\ \sum_{i=1}^{N}(\text{ESTAR}_i - f_i)^2}{N}}$$

T ESTAR values, ESTAR$_i$, are given for comparison and predicted by the semi-empirical model, eq(1), and are denoted by f$_i$.

3. Results and Discussion

Rigorous comparisons of the electron ranges f(T) calculated using semi-empirically eq(1) were made to NIST ESTAR benchmark data for both ICRP brain tissue and eye lens tissues were investigated over the electricity interval from 100 to 550 MeV [13]. The CSDA range of electrons in brain tissue and eye lens become calculated and as compared with the corresponding reference data to evaluate the accuracy of the proposed semi-empirical model.

3.1. The semi-empirical predictions (f) eq(1) compared with the ICRP brain tissue standard NIST ESTAR values (Estar) are summarized in Table 1

Table 1: Electron CSDA range in ICRP brain tissue in g/cm^2 over energy range 100–550 MeV

Energy T (MeV)	ESTAR Range Estar (g/cm^2)	Semi-Empirical f (g/cm^2)	Relative Error (%)
100	32.80	33.10	0.92%
125	37.89	38.51	1.65%
150	42.37	43.15	1.84%
175	46.38	47.23	1.83%
200	50.00	50.88	1.77%
250	56.33	57.25	1.64%
300	61.75	62.71	1.55%
350	66.48	67.51	1.54%
400	70.68	71.81	1.59%
450	74.46	75.71	1.68%
500	77.88	79.30	1.82%
550	81.02	82.62	1.98%

3.2. Electron CSDA Range in Eye Lens Tissue

Table 2 lists the comparative performance data for ICRP eye lens tissue [14]:

Table 2: Electron CSDA range in ICRP eye lens tissue in g/cm^2 over energy range 100–550 MeV

Energy T (MeV)	ESTAR Range Estar (g/cm^2)	Semi-Empirical f (g/cm^2)	Relative Error (%)
100	33.30	33.10	0.59%
125	38.49	38.51	0.06%
150	43.06	43.15	0.21%
175	47.14	47.23	0.19%
200	50.83	50.88	0.11%
250	57.30	57.25	0.08%
300	62.83	62.71	0.19%
350	67.66	67.51	0.23%
400	71.95	71.81	0.20%
450	75.81	75.71	0.13%
500	79.31	79.30	0.01%
550	82.52	82.62	0.12%

3.3. Statistical Indicators Summary

Table 3 summarizes the statistical evaluation parameters for both tissue types [15]:

Table 3: Statistical evaluation metrics comparing semi-empirical eq(1) predictions with NIST ESTAR data

Tissue Type	Correlation R	R^2 Value	Mean Error (MAPE)	RMSE (g/cm^2)
Eye Lens (ICRP)	0.99998	0.99996	0.18%	0.108
Brain Tissue (ICRP)	0.99998	0.99996	1.65%	1.034

3.4. Graphical Analysis and Comparisons

Figures 1, 2, and 3 illustrate the individual and combined comparative plots between the semi-empirical eq(1) curve and NIST ESTAR benchmark points across the energy range (100–550 MeV) [16–18]:

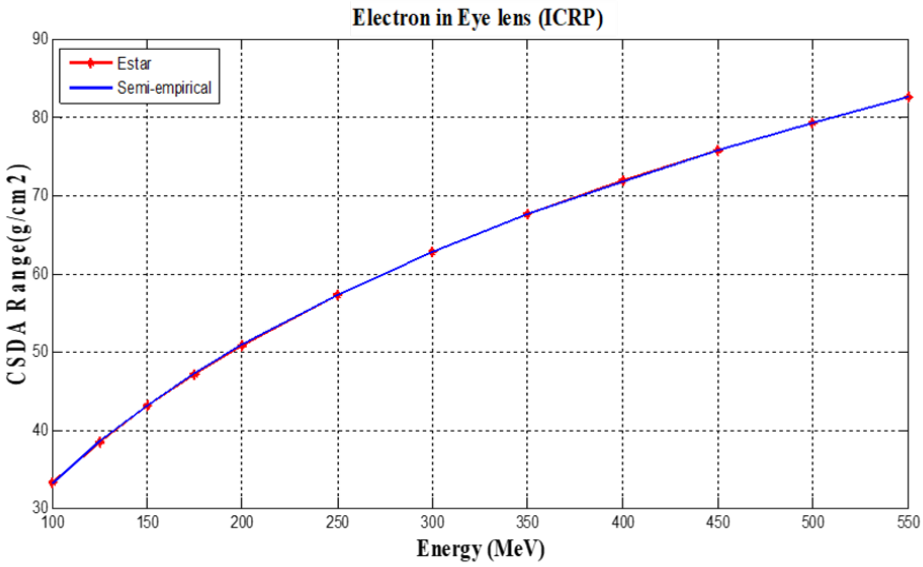


Figure 1: Electron CSDA range (g/cm^2) versus kinetic energy T (MeV) in ICRP Eye Lens tissue

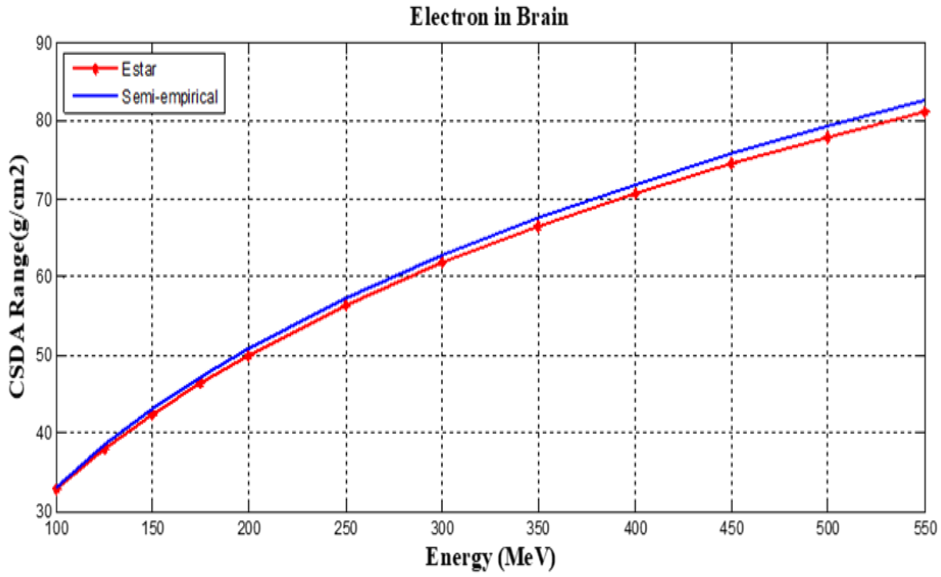


Figure 2: Electron CSDA range (g/cm^2) versus kinetic energy T (MeV) in ICRP Brain tissue

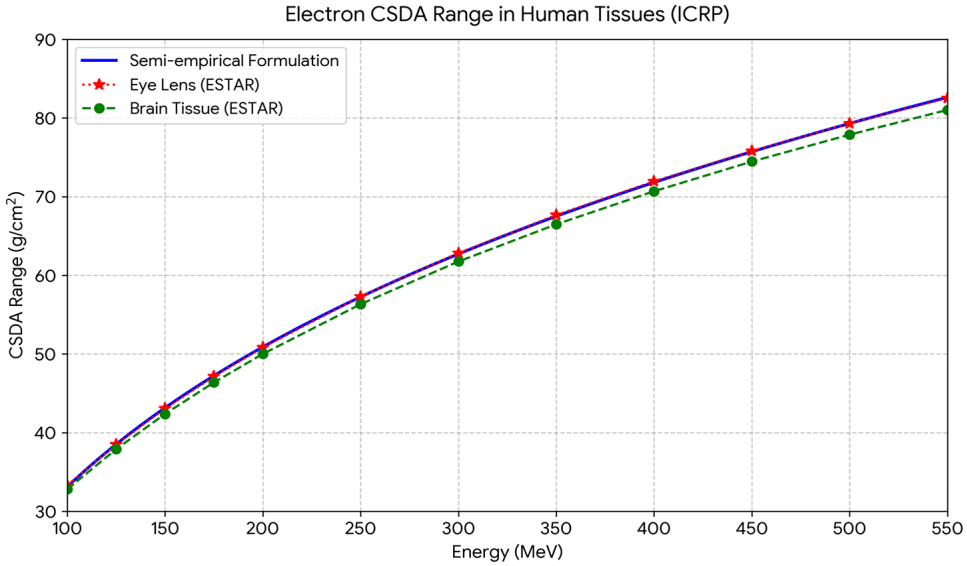


Figure 3: Combined plot of electron CSDA range (g/cm^2) versus kinetic energy T (MeV) comparing ICRP Eye Lens and Brain tissue ranges with the generalized semi-empirical curve

4. Conclusion

Finally, this study proposed a semi-empirical generalized eq(1) power-law equation with parameters a = 34.12, b = 0.2348 and c = -67.5, to accurately calculate the electron CSDA range in brain and eye lens tissues, within the high energy range of 100 to 550MeV. The rigorously validated result, using NIST ESTAR benchmark data, showed a remarkable correlation of R = 0.99998 (R^2 = 0.99996) for both tissues. The mean absolute percentage error (MAPE) values were also very low for eye lens (0.177%) and brain tissue (1.652%), while the RMSE values were 0.108 g/cm^2 and 1.034 g/cm^2, respectively. Most importantly, these results indicate the single semi-empirical mathematical equation is versatile and can be reliably used for all human soft biological tissues in radiation therapy treatment planning, providing an immediate, highly accurate analytical solution to Monte Carlo simulation.

Article Information

Ethical Approval: Not applicable.

Data Availability: The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Conflict of Interest: The author declare that they have no conflict of interest.

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